

were detected and triply charged atoms of oxygen, nitrogen, and carbon. The compound OH_4 was also detected carrying invariably a double charge.

I am much indebted to the Royal Society for the loan of the radium; to Sir James Dewar for placing at my disposal large supplies of the residues of liquid air, and to Mr. E. Everett for the assistance he has given me in making the experiments.

On the Scattering of β -particles.

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(Communicated by Prof. Sir E. Rutherford, F.R.S.—Received April 7, 1922.)

§ 1. *Introduction.*

An account has recently been published* by Dr. J. A. Crowther and the writer, of some experiments on the scattering of β -rays by thin sheets of matter. The angles of deflection investigated ranged from 4° to 8° . The main results obtained may be summarised as follows:—

(1) The scattering observed was “single,” *i.e.*, due to one collision with an atomic nucleus.

(2) The law of force governing such collisions appeared to be that of the inverse square.

(3) The single scattering theory due to Sir Ernest Rutherford requires the scattering power of an element to be proportional to the square of its nuclear charge. This relation was found to hold only in the case of the elements carbon and aluminium, if the nuclear charge consist of as many positive units as the atomic number of the element.

(4) For the elements copper, silver, gold and platinum, a comparison with previous results extending the range of the deflection angle to 20° † showed that a rapid increase in the scattering power of these elements appeared to take place as the thickness and angle of deflection were increased.

The experiments to be described were instituted to test the truth of (4) and to confirm the other conclusions under the somewhat easier experimental conditions prevailing at larger angles of deflection. They were carried out at from 12° to 25° .

* Crowther and Schonland, ‘Roy. Soc. Proc.,’ A, vol. 100 (1922).

† Crowther, ‘Roy. Soc. Proc.,’ A, vol. 84 (1910).

A further conclusion reached in the previous paper was that the numerical value of the scattering observed was 2.9 times greater than that to be expected on the theory mentioned. This conclusion, which does not affect those given above, was also examined and found to be incorrect.

In a recent paper Geiger* has given results of an investigation of this problem by the photographic method, which confirm (2), (3) and (4).

§ 2. *Experimental Arrangement.*

The apparatus used was similar to that described in the previous paper, modified however to deal with angles of deflection from 12° to 25° . The angular divergence of the β -ray beam was proportionately increased, giving an ionisation effect which was some ten times greater than before.

This made easy an examination of the sources of error in the arrangement, directed towards the following points:—

(1) A possible effect of the stray magnetic field on the scattered particles. This was found to be negligible, the magnetic shielding being ample.

(2) A possible difference between the velocity of the β -rays examined and that calculated from the bending in the magnetic field. A proportion of the beam might consist of scattered and secondary β -particles from the walls of the bending chamber and firing tube. Such rays of sufficient penetrating power to pass into the measuring chamber were found not to constitute as much as 1 per cent. of the beam.

(3) An effect due to the divergence of the beam. This was found to be very important.

§ 3. *The Error in Absolute Value due to Divergence of the Beam.*

The usual arrangement of the experiments is shown in diagrammatic form in fig. 1.

The shaded area represents the beam of rays, PQ the circular aperture, and AB the scattering foil. Rays incident on the foil in the direction OA will, after scattering, be faced with an angular aperture which is not ϕ , but varies from θ_1 to θ_2 with their scattered direction. If we suppose the beam to be annular in cross-section, *i.e.*, concentrated at the edges of the cone OA, OB, integration shows that the observed scattering will be greater than that calculated, by the factor

$$\gamma = \frac{\sec^2 \phi}{(1 - \alpha^2/\phi^2)^2}, \quad \text{where } \alpha = \frac{1}{2} \text{ angle of divergence of beam,} \\ \phi = \frac{1}{2} \text{ angular aperture,} \\ = 1.7 \text{ for } \alpha/\phi = 0.4 \text{ and } \phi = 20^\circ.$$

* Geiger, 'Zeitschrift für Physik,' vol. 6 (1921).

If, however, the beam is supposed uniform over its cross-section, the value of γ is only 1.1.

The experimental examination of the value of γ was effected by reducing the divergence of the beam in steps until it had the form shown in fig. 2.

FIG. 1.

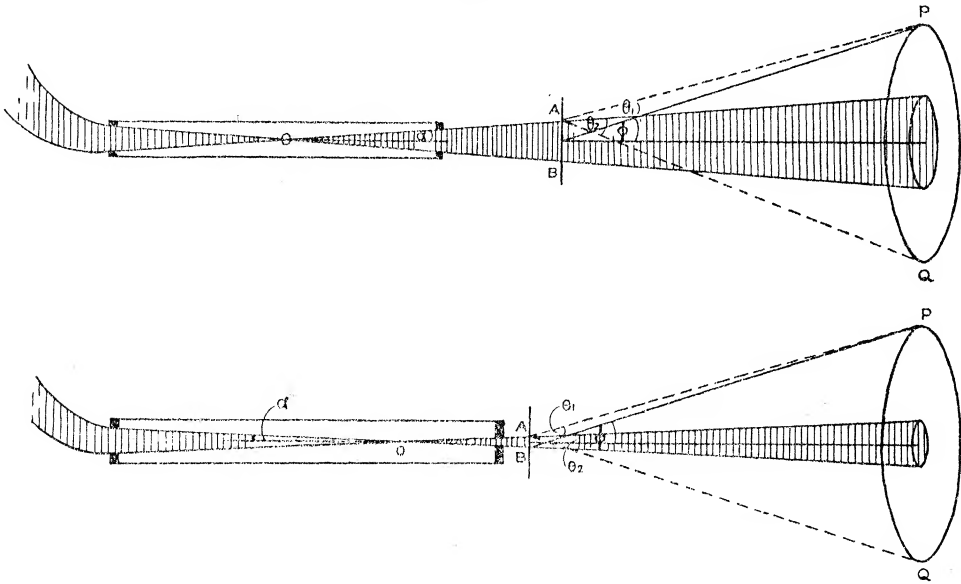


FIG. 2.

The effect produced on the observed scattering is shown in fig. 3. Curve I corresponds to fig. 1, curve III to fig. 2, and curve II to an intermediate arrangement. It is seen that the scattering (I/I_0) is decreased 1.8 times by

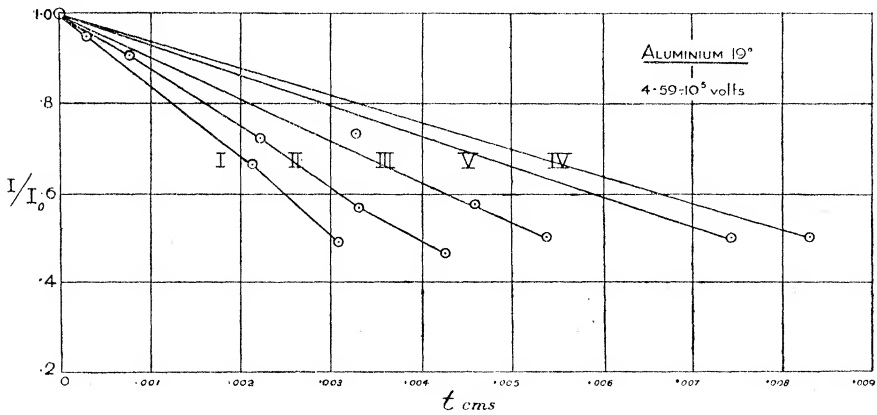


FIG. 3.

changing the conditions from fig. 1 to fig. 2. Curve IV is the calculated scattering curve, and curve V is obtained from this by an alteration of 10 per cent. This is warranted by the consideration that the effects of absorption and reflection of rays by the aluminium window closing the scattering chamber vary as $\sec^2\phi$, and ϕ is here about 20° .

The final curves, III and V, differ by 28 per cent. in the value of I/I_0 for a given thickness. A precisely similar set of curves was obtained for $\phi = 25^\circ$, the difference here being 26 per cent. In view of small residual errors due to the divergence of the beam and other causes, the agreement between theory and experiment is as near as could be expected from the experimental arrangement. The results give a value for the nuclear charge of aluminium of about 15 units, instead of the atomic number 13.

Photographs of the beam were taken to investigate the distribution of the rays over the cross-section. They show a decided tendency towards an annular distribution, which, as has been seen, would explain the effect observed.

§ 4. *The Variation of Scattering with Angle of Deflection.*

On the single nuclear scattering theory applied to β -particles, the fraction scattered through an angle greater than ϕ

$$\rho = \frac{\pi}{4} n t b^2 \cot^2 \frac{\phi}{2} f^2(\phi, \beta). \quad (1)$$

Here n = no. of atoms p. u. vol., and is const. for Au, Ag, and Al,

$b = \frac{Ne^2}{[\text{Energy}]}$, the energy being expressed by the Planck-Einstein formula,

$f^2(\phi, \beta)$ is the relativity correction (*vide* Sec. 6).

At constant energy (1) reduces to

$$\rho = \text{const.} \times \frac{tN^2}{\phi^2}, \text{ since } f^2(\phi, \beta) \text{ varies slowly with angle } \phi,$$

or if $t = t_m$ when $\rho = \frac{1}{2}$,

$$\frac{\phi^2}{t_m} :: N^2. \quad (2)$$

Thus the curve for each element relating ϕ and t_m should be a parabola with latus rectum proportional to the square of its atomic number. Fig. 4 shows that for aluminium this parabolic relation is followed throughout the range examined. For heavier elements the parabola becomes a straight line as the thickness t_m diminishes.

Curves similar to those above have been recently found by Geiger,*

* Geiger, *loc. cit.*

using the photographic method. Taking the parabolic portions of the curves the Table below shows that relation (2) is approximately fulfilled.

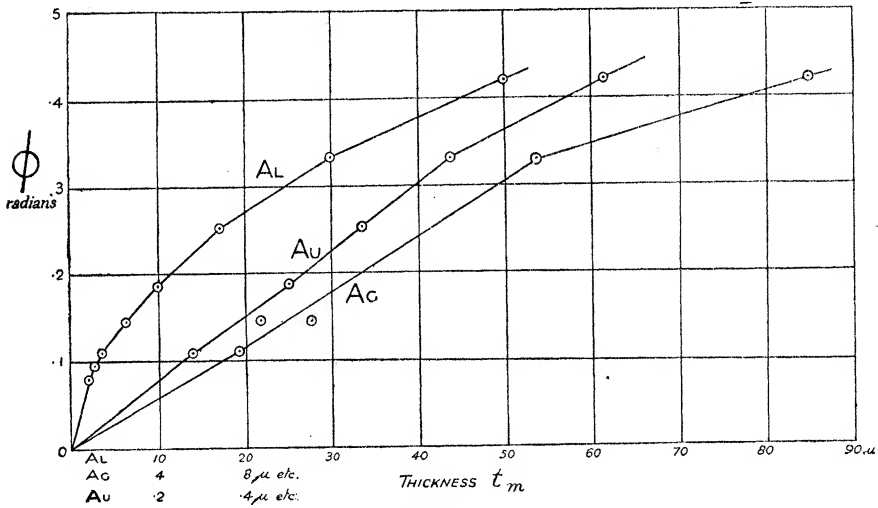


FIG. 4.

Element.	t_m for $\phi = 0.42$ radians.	$t_m \times N^2$.
	cm.	
Al	0.0050	84
Ag	0.00034	75
Au	0.000124	78

The behaviour of aluminium is shown by the constancy of $\phi/\sqrt{(t_m)}$ in the Table below.

ϕ .	$\phi/\sqrt{(t_m)}$.	
0		
5	5.90	} Published in previous paper.
6	5.95	
8	5.92	
10.5	5.93	
14.3	6.04	
18.9	6.04	
24.0	5.88	

For gold and silver, after a certain critical value for t_m , ϕ/t_m is found to be approximately constant, as fig. 4 shows. An explanation of this apparent anomaly follows.

§ 5. *The Screening Effect of Electrons in the K and L Rings.*

The results given show that as the thickness is reduced, the scattering of β -rays by the elements Au, Ag and Cu falls *below* the theoretical value as soon as a certain critical thickness is passed. This thickness has not been reached for aluminium or carbon.

The falling off in the scattering power is progressive and continuous. For gold the reduction has been pushed to one-third of the original scattering power.

The phenomenon would be explicable if there were reason to suppose that as the thickness t_m of the foil is diminished by reducing the observed angle of deflection ϕ , the effective nuclear charge responsible for scattering is in some way diminished too. Such an explanation is offered by the screening action of the electrons surrounding the nucleus in the K and L rings.

If t_m is the thickness, d_m the distance within which one-half of the incident particles come to some one nucleus, n the number of atoms per unit volume,

$$\pi n t_m d_m^2 = \frac{1}{2},$$

or

$$d_m :: \frac{1}{\sqrt{(t_m)}}.$$

Thus as the thickness is diminished the distance of approach which corresponds to t_m increases. Provided it can be shown that this increase of d_m brings the orbit of the β -particle into the region occupied by the electrons, the decrease of the effective nuclear charge receives an explanation. In spite of the fact that in these experiments we are dealing with particles whose distances of approach to the various nuclei that deflect them are not all equal, the distance d_m and the information about the nuclear charge derived from it are perfectly definite. For d_m is that distance of approach to a nucleus for which the particle is *just* deflected through an angle ϕ .

The experiments show that at all the values of d_m examined the nuclear charge for aluminium is constant and equal to its atomic number within the error of experiment. For gold, the nuclear charge varies from 79 to 51 units as d_m is increased. For silver similar results hold, but the constant and correct value of the nuclear charge is reached before d_m has the lowest value examined.

If, as has been suggested, the electrons in the "rings" surrounding the nucleus produce this reduction in the effective nuclear charge, it must be shown that the distances d_m for the various elements are such as to bring the orbits of the β -particles under the influence of these electrons to the extent

required. This means that in all the experiments with aluminium the β -particles penetrate inside the K ring of that element, while in the case of gold these orbits are described with the electrons of the K ring, and other outside electrons, intervening between the β -particles and the nucleus. The Table below shows that this is the case. The maximum and minimum values of d_m given correspond to angles of deflection of 6° and 25° . They should be compared with the calculated values of the "radii" of the rings. The last two columns give the maximum reduction in the atomic nuclear charge n_{obs} (this is found at d_m max.), and a calculated value of n which will be discussed.

Fig. 5 gives a representation of the tabulated results, drawn to scale.

Element.	d_m (min.).	d_m (max.).	r_L .	r_K .	$n_{\text{obs.}}$	$n_{\text{calc.}}$
	cm.	cm.	cm.	cm.		
Al	$2.3 \cdot 10^{-11}$	$9.0 \cdot 10^{-11}$	—	$5.4 \cdot 10^{-10}$	0	0
Ag	$9.0 \cdot 10^{-11}$	$2.0 \cdot 10^{-10}$	$5.0 \cdot 10^{-10}$	$1.0 \cdot 10^{-10}$	16	3 to 16
Au	$1.5 \cdot 10^{-10}$	$3.0 \cdot 10^{-10}$	$3.4 \cdot 10^{-10}$	$7.0 \cdot 10^{-11}$	26	16

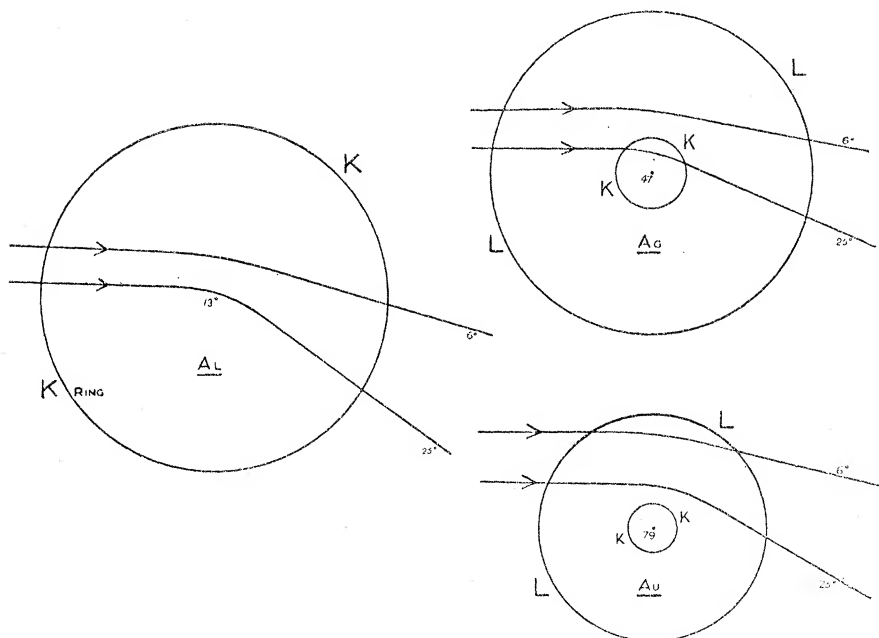


FIG. 5.—1 cm. = 10^{-10} cms.

The calculations of r_L and r_K as well as n_{calc} have been made as follows. The Bohr theory of the hydrogen atom can be applied to this purpose in the case of more complex atoms only indirectly. This is due to the effect

mentioned, namely the screening of the nucleus by successive rings, and to the mutual repulsion of the electrons themselves. Any deduction of the radii based entirely upon theory has less weight than the results to be obtained from X-ray absorption spectra. The method used for getting the figures tabulated above is:—

(a) Calculate N_{eff} , the effective nuclear charge at the K or L ring from the equation

$$N_{\text{eff}}^2 \cdot \nu_{\text{H}} = \nu_{\text{E}} \text{ where } \nu_{\text{H}} \text{ is the frequency of the } \begin{cases} \text{first Bohr ring for} \\ \text{second Hydrogen,} \end{cases}$$

and ν_{E} is the maximum absorption frequency of the $\begin{cases} \text{K X-ray spectrum of the} \\ \text{L element considered.} \end{cases}$

Then $n_{\text{calc}} = N - N_{\text{eff}}$.

The following approximate values for n_{calc} hold for elements between Al and Au.

	K ring.	L ring.	M ring.
$n_{\text{calc.}}$	3	16	30

(b) Consider the electrons as revolving in simple Bohr orbits round the nuclear charge N_{eff} .

Then

$$r_{\text{K}} = \frac{0.538 \cdot 10^{-8}}{N_{\text{K eff}}} \text{ cms.,}$$

$$r_{\text{L}} = \frac{4 \times 0.538 \cdot 10^{-8}}{N_{\text{L eff}}} \text{ cms.,}$$

$N_{\text{K eff}}$ and $N_{\text{L eff}}$ being the values of N_{eff} at the K and L rings.

The assignment of definite values to these "radii" is, of course, to some extent artificial, for on Bohr's later views the electrons have complicated orbits round the nucleus. Their distribution will thus resemble a diffuse cloud more than a discontinuous set of rings. The calculation of n given is more likely to be correct than any deduction at present possible which is based on the theoretical distribution and orbits of the electrons. That n_{calc} is likely to be less than n_{obs} is clear from the fact that only at its *nearest* point of approach to the nucleus does the β -particle come under as large an effective nuclear charge as $N - n_{\text{calc}}$. Thus the higher values of n_{obs} are to be expected.

It is interesting to compare these results with some early experiments of Geiger* on the scattering of α -particles by thin foils of gold, where the same effect is shown. The table gives his experimental figures, the most probable angle of deflection (which is $\phi/1.18$) being termed λ . p is the correction to

* Geiger, 'Roy. Soc. Proc.,' A, vol. 81, p. 174 (1908).

λ for loss of velocity in passing through the foil, and in the final column is $\lambda p/\sqrt{t}$ which should be constant. For small thicknesses this is not true and the critical thickness is found to be about the same as in the present experiments, namely, that at which the most probable distance of approach reaches the neighbourhood of the L ring.

No. of foils t .	Stopping power.	λ .	p .	$\lambda p/\sqrt{t} \times 10$.
	cm. (air).	'		
1	0.04	10	1.0	1.00
2	0.08	23	1.0	1.62
4	0.15	36	1.0	1.80
8	0.29	49	0.985	1.78
12	0.46	70	0.98	1.96
20	0.76	100	0.97	2.14
30	1.14	140	0.94	2.40

} Outside region of L ring.

§ 6. *The Orbit of a β -particle in Collision with a Positive Nucleus.*

The following deduction of a "relativity correction" to the original Rutherford theory of scattering of α - and β -particles is based on a paper* on β -ray absorption by Mr. C. G. Darwin and a further application of it to the present problem which he very kindly worked out for me.

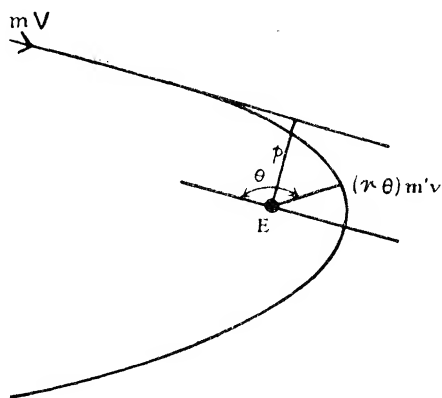


FIG. 6.

Let V = initial velocity of the particle,
 m = initial mass,
 v = velocity at pt r , θ ,
 m' = mass at pt r , θ .

Then $m' = \frac{m_0}{(1-v^2/c^2)^{\frac{1}{2}}}$ and $m = \frac{m_0}{(1-V^2/c^2)^{\frac{1}{2}}}$.

* C. G. Darwin, 'Phil. Mag.', vol. 25, p. 201 (1913).

From the conservation of angular momentum

$$m'r^2\dot{\theta} = mpV,$$

where p is the perpendicular from the nucleus of charge E on the original direction of motion,

or
$$\frac{r^2\dot{\theta}}{(1-v^2/c^2)^{\frac{1}{2}}} = \frac{pV}{(1-V^2/c^2)^{\frac{1}{2}}}. \quad (3)$$

From the conservation of energy

$$m_0c^2 \left\{ \frac{1}{\sqrt{(1-v^2/c^2)}} - 1 \right\} = m_0c^2 \left\{ \frac{1}{\sqrt{(1-V^2/c^2)}} - 1 \right\} + \frac{Ee}{r},$$

or
$$\frac{1}{\sqrt{(1-v^2/c^2)}} = \frac{1}{\sqrt{(1-V^2/c^2)}} + \frac{Ee}{m_0c^2} \cdot \frac{1}{r}. \quad (4)$$

From (3) and (4), eliminating v

$$r^2\dot{\theta} = \frac{pc \sin \beta}{1 + (a/r) \cos \beta}, \quad \begin{cases} \sin \beta = V/c \\ a = Ee/m_0c^2 \end{cases}$$

From (4)
$$\dot{r}^2 + r^2\dot{\theta}^2 = c^2 \left\{ 1 - \frac{\cos^2 \beta}{(1 + (a/r) \cos \beta)^2} \right\}.$$

By integration from these last two equations, the orbit is

$$\frac{p\gamma}{r} = \sin \gamma\theta + (1 - \cos \gamma\theta) \frac{cp_0}{V\gamma p},^*$$

where

$$\begin{aligned} p_0 &= a \cot \beta, \\ \gamma^2 &= 1 - p_0^2/p^2. \end{aligned}$$

This orbit is described in the paper quoted. In the cases to which it is here applied we do not need to consider the spiral form it may assume. Nor does loss of energy by radiation play any part in these collisions.

Writing $p = p_0 \operatorname{cosec} \psi$ we find $\gamma = \cos \psi$ and the orbit is

$$\frac{p\gamma}{r} = \sin (\theta \cos \psi) + [1 - \cos (\theta \cos \psi)] \frac{c}{V} \tan \psi.$$

If the deflection is ϕ , $\pi + \phi$ is the angle between the asymptotes and $\frac{1}{2}(\pi + \phi)$ is the angle of minimum radius.

Thus $\theta = \frac{1}{2}(\pi + \phi)$ when $\cos (\theta \cos \psi) + \sin (\theta \cos \psi) \frac{c}{V} \tan \psi = 0$,

and

$$V/c \cot \psi = \tan \{ \pi - (\cos \psi) \frac{1}{2}(\phi + \pi) \}.$$

From the solution of this ψ can be found in terms of ϕ and V/c . It is

* Cf. Sommerfeld's treatment of the elliptical orbit in the theory of the fine-structure of spectral lines, 'Atombau u. Spektrallinien,' chap. 5, § 2.

independent of E . As shown in the previous paper* the correction in the Rutherford scattering formula may be written

$$f^2(\phi, \beta) = \frac{\operatorname{cosec}^2 \psi}{\left\{ \frac{\beta \cot \phi/2}{2(1 - \sqrt{1 - \beta^2})} \right\}^2}.$$

If the energy is originally written as $\frac{1}{2}mV^2$, where $m = m_0(1 - \beta^2)^{-\frac{1}{2}}$, the correction is

$$\frac{\operatorname{cosec}^2 \psi}{1/\beta^2(1 - \beta^2) \cot^2 \phi/2}.$$

Some values of $f^2(\phi, \beta)$ are given below—

$V/c = \beta$.	ϕ .	$f^2(\phi, \beta)$.
0·851	0	
	2·4	1·79
	6	1·81
	8	1·92
	19	2·09
	25	2·25
0·708		1·59
0·851		2·09
0·924	19	2·66
0·944		2·91

The last part of the third column shows that measurements on the variation of scattering with velocity should deviate considerably from the relation obtaining on the simpler theory. These will now be considered.

The Variation of Scattering with Energy.

Equation (1) $\rho = \frac{1}{4}\pi nt b^2 \cot^2 \frac{1}{2}\phi f^2(\phi, \beta)$ for this case can be written in the form

$$t_m :: \frac{[\text{Energy}]^2}{f^2(\phi, \beta)} \text{ if the energy is expressed as } m_0 c^2 \left(\frac{1}{\sqrt{1 - \beta^2}} - 1 \right),$$

$$\text{or} \quad \frac{[\text{Energy}]^2}{f^2(\phi, \beta) t_m} = \text{const.} \quad (3)$$

Column 8 of the Table below shows that for the measurements made on aluminium, and given in fig. 7, this relation holds within the error of experiment. This is strong evidence for the correctness of the argument in the preceding section, as may be seen by comparing with column 6, in which the relativity correction is omitted. The latter omission, however, amounts to writing for equation (4)

$$\frac{1}{2} m' v^2 = \frac{Ee}{r} + m_0 c^2 \left\{ \frac{1}{\sqrt{1 - V^2/c^2}} - 1 \right\}$$

which is indefensible.

* *Ibid.*, p. 540.

It is, however, interesting to notice that, if the energy of the particle is written as $\frac{1}{2} m_0 (1 - \beta^2)^{-\frac{1}{2}} v^2$, the relation energy^2/t_m (column 7) is very nearly constant. This explains why previous observers* have found the formula mentioned to fit their results. For, under the conditions of these and previous experiments,

$$\frac{E^2}{f^2(\phi, \beta)} = E' \text{ within about 20 per cent.,}$$

where $E = m_0 c^2 \left\{ \frac{1}{\sqrt{(1 - \beta^2)}} - 1 \right\}$ and $E' = \frac{1}{2} m_0 (1 - \beta^2)^{-\frac{1}{2}} V^2$.

The conclusion, which must be regarded as accidental, that if the complete relativity expressions for the energy and the orbit are used, the result numerically is almost the same as in the earlier and simpler form, only holds approximately and should fail with slower rays and larger angles of deflection.

Aluminium.

1.	2.	3.	4.	5.	6.	7.	8.
H ρ .	E volts $\times 10^{-5}$.	E' volts $\times 10^{-5}$.	$f^2(\phi, \beta)$.	t_m .	E^2/t_m .	E'^2/t_m .	$E^2/f^2(\phi, \beta) t_m$.
1694	2.11	1.80	1.59	0.0010	$4.45 \cdot 10^{-6}$	$3.23 \cdot 10^{-6}$	$2.80 \cdot 10^{-6}$
2742	4.59	3.48	2.09	0.0038	5.55	3.19	2.65
4101	8.23	5.68	2.66	0.0090	7.52	3.59	2.82
4842	10.3	6.80	2.91	0.0124	8.57	3.73	2.93

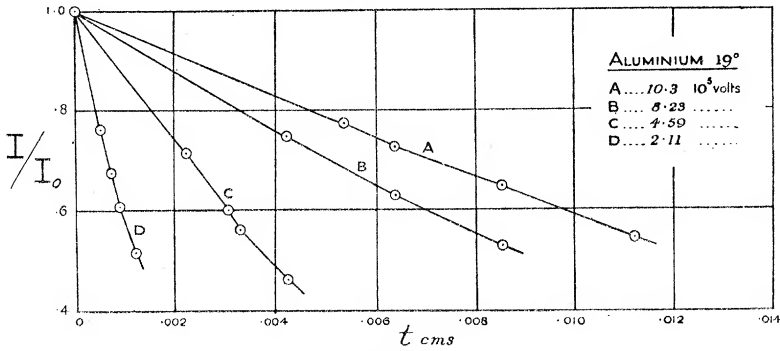


FIG. 7.

A similar variation with velocity was found in a number of experiments made with gold and platinum, but owing to the complication discussed in Section 4, the results are not as simple as for aluminium. A number of

* Crowther, 'Roy. Soc. Proc.' A, vol. 84 (1910); Geiger, 'Zeit. für Physik,' vol. 6 (1921).

careful determinations gave values for the scattering by platinum which, suitably compared, agreed within a few per cent. with the figures for gold.

§ 6. *Summary of Results.*

The experiments described, together with those of Crowther and the writer, lead to the following conclusions:—

1. The scattering of β -particles observed is a single scattering phenomenon, due to collisions with the nuclei of the atoms encountered.

2. The relations following from Rutherford's theory of single scattering are in general obeyed, though a correction due to the increase of mass with velocity is necessary in the case of the β -particle.

3. Evidence for this correction has been found in the variation of scattering with velocity.

4. The anomalous behaviour of the heavier elements described in the previous paper has been traced, and an explanation based upon the screening action of the electrons surrounding the nucleus appears to be satisfactory.

5. The sources of error in the form of apparatus used have been examined, and absolute numerical values for the scattering found which are in satisfactory agreement with the theory. It may be added that (4) offers direct evidence for the approximate position and distribution of the electrons immediately surrounding the nucleus. From (3) and § 5 it is concluded that the law of force in these collisions is that of the inverse square, within from 10^{-11} or 10^{-10} cms. of the nucleus of an atom.

My grateful thanks are due to Sir Ernest Rutherford for his encouragement and guidance in this research. I am much indebted to Mr. G. R. Crowe for his help in preparing the emanation tubes.
